Extending a Novel Approach for Clustering Time-Series

Treating Multivariate Time-Series as Polygonal Curves

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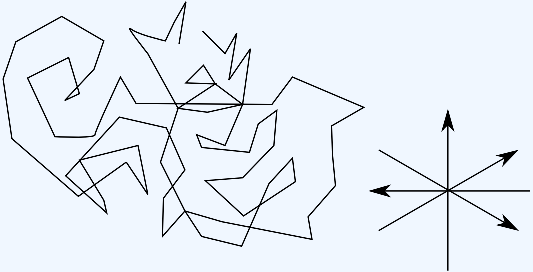

Motivation

Fréchet distance is **building block** in many (machine learning) applications

- morphing
- protein structure alignment
- handwriting recognition
- **clustering of time-series** (weather, large physical experiments, stock...)
  - ☞ compensate different sampling-rates and inhomogeneous lengths by only comparing the "shape"
    - Driemel et al. (SODA 2015): Clustering time-series under the Fréchet distance (univariate)
    - (NeurIPS 2019) this work (multivariate)

- sum- or average-based distance? **intractable** for high dimensions (see Efrat (2007): Curve Matching, Time Warping ...)
- bottle-neck distance: **Fréchet distance** a.k.a. "dog-man distance"

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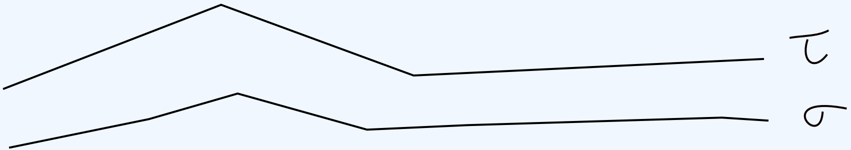

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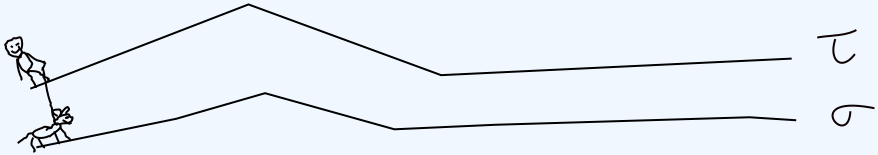

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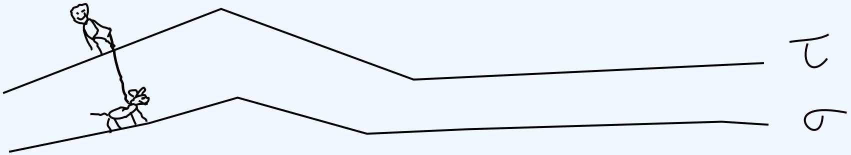

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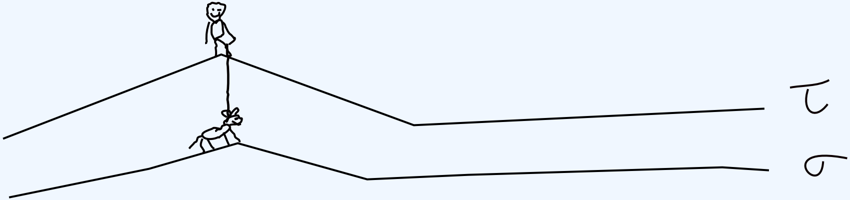

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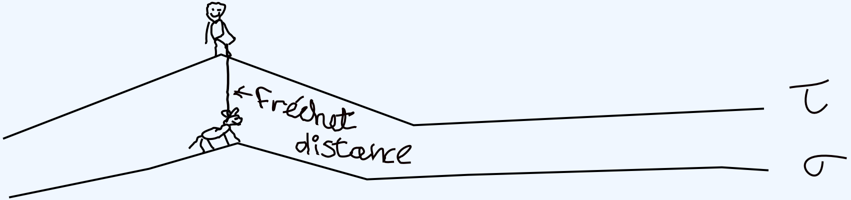

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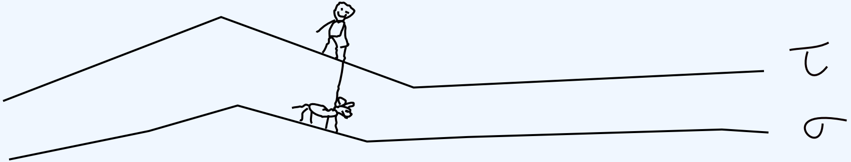

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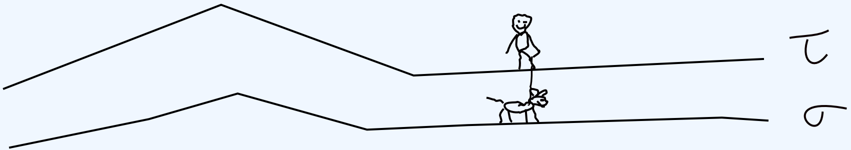

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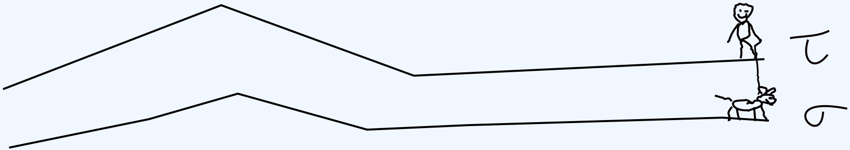
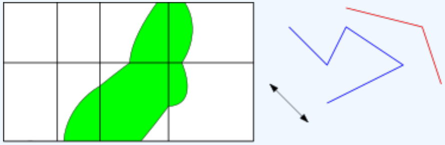


- **Alt-Godau Algorithm** (1995: Computing the Fréchet distance between two Polygonal Curves)
  - Running-time $\mathcal{O}(d \cdot m^2 \log(m))$
- There is no algorithm with running-time $\mathcal{O}(m^{2-\delta})$, for any $\delta > 0$, unless SETH' fails (Bringmann 2014: Why walking the dog takes time...)



What Happens in the Big Data Regime?

- n: (large) number of curves to cluster
- high complexity, say $m \in \Omega(n)$
- high-dimensional, say $d \in \Omega(n)$

$\rightarrow$ Running-time super-cubic in n in the worst case
  - **But** quadratic running-time is already considered intractable on Big Data

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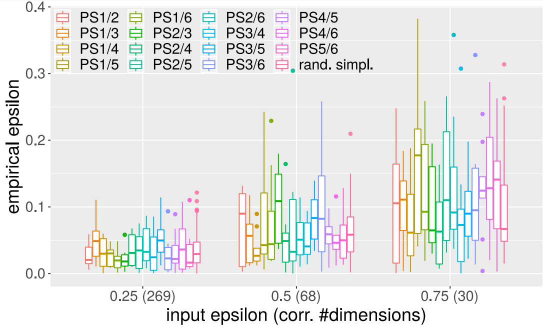
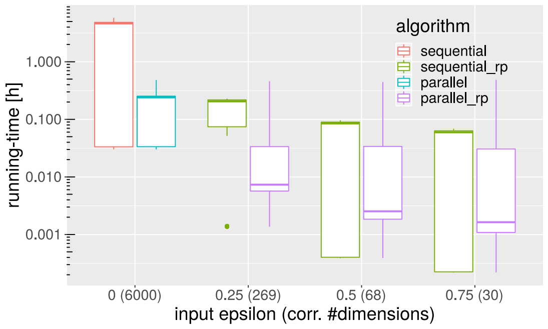


(a) Quality



(b) Running-time